

Prva školska zadaća - ISPRAVAK
(WD)

1. Skrati razlomke

A. $\frac{a^{2n} - 4a^n + 4}{a^n - 2}$ B. $\frac{x^{2n} + 6x^n + 9}{x^n + 3}$

1. Rješenje:

a)

$$\begin{aligned}\frac{a^{2n} - 4a^n + 4}{a^n - 2} &= \frac{(a^n)^2 - 2 \cdot a^n \cdot 2 + 2^2}{a^n - 2} \\ &= \frac{(a^n - 2)^2}{a^n - 2} = a^n - 2\end{aligned}$$

b)

$$\begin{aligned}\frac{x^{2n} + 6x^n + 9}{x^n + 3} &= \frac{(x^n)^2 + 2 \cdot x^n \cdot 3 + 3^2}{x^n + 3} \\ &= \frac{(x^n + 3)^2}{x^n + 3} = x^n + 3\end{aligned}$$

2. Za polinome

A: $P(x) = 3x^2 - 2x - 2$ i $Q(x) = -x^2 - 2x + 1$

B. $P(x) = -2x^2 + 3x - 2$ i $Q(x) = 2x^2 - 2x - 1$

Example 1 *odredi*

$$P(x) + Q(x), P(x) - Q(x), P(x) \cdot Q(x)$$

Rješenje:

A.

$$\begin{aligned}P(x) + Q(x) &= 3x^2 - 2x - 2 - x^2 - 2x + 1 = 2x^2 - 4x - 1 \\ P(x) - Q(x) &= 3x^2 - 2x - 2 - (-x^2 - 2x + 1) \\ &= 3x^2 - 2x - 2 + x^2 + 2x - 1 = 4x^2 - 3 \\ P(x) \cdot Q(x) &= (3x^2 - 2x - 2)(-x^2 - 2x + 1) \\ &= -3x^4 - 6x^3 + 3x^2 + 2x^3 + 4x^2 - 2x + 2x^2 + 4x - 2 \\ &= -3x^4 - 4x^3 + 9x^2 + 2x - 2\end{aligned}$$

B.

$$\begin{aligned}P(x) + Q(x) &= -2x^2 + 3x - 2 + 2x^2 - 2x - 1 = x - 3 \\P(x) - Q(x) &= -2x^2 + 3x - 2 - (2x^2 - 2x - 1) \\&= -2x^2 + 3x - 2 - 2x^2 + 2x + 1 = -4x^2 + 5x - 1 \\P(x) \cdot Q(x) &= (-2x^2 + 3x - 2)(2x^2 - 2x - 1) \\&= -4x^4 + 4x^3 + 2x^2 + 6x^3 - 6x^2 - 3x - 4x^2 + 4x + 2 \\&= -4x^4 + 10x^3 - 8x^2 + x + 2\end{aligned}$$

3. Kvadriraj binome

$$\begin{aligned}\text{A. a)} \quad & \left(\frac{1}{2}a - \frac{3}{4}b\right)^2; & \text{b)} \quad 3(a+2)^2 - 27 \\ \text{B. a)} \quad & \left(\frac{2}{3}x - \frac{3}{2}y\right)^2; & \text{b)} \quad 2(x+3)^2 - 32\end{aligned}$$

Rješenje: **A.**

$$\left(\frac{1}{2}a - \frac{3}{4}b\right)^2 = \frac{1}{4}a^2 - \frac{3}{4}ab + \frac{9}{16}b^2$$

$$\begin{aligned}3(a+2)^2 - 27 &= 3[(a+2)^2 - 9] = 3(a+2-3)(a+2+3) \\&= 3(a-1)(a+5)\end{aligned}$$

B.

$$\left(\frac{2}{3}x - \frac{3}{2}y\right)^2 = \frac{4}{9}x^2 - 2xy + \frac{9}{4}y^2$$

$$\begin{aligned}2(x+3)^2 - 32 &= 2[(x+3)^2 - 16] = 2(x+3-4)(x+3+4) \\&= 2(x-1)(x+7)\end{aligned}$$

4. Kuiraj binome

$$\begin{aligned}\text{A. a)} \quad & (2a+b)^3; & \text{b)} \quad 8x^3 - 27y^3 \\ \text{B. a)} \quad & (x-2y)^3; & \text{b)} \quad 27a^3 + 8b^3\end{aligned}$$

Rješenje:

A.

$$(2a - b)^3 = 8a^3 - 12a^2b + 6ab^2 - b^3$$
$$8x^3 - 27y^3 = (2x - 3y)(4x^2 - 6xy + 9y^2)$$

B.

$$(x - 2y)^3 = x^3 - 6x^2y + 12xy^2 - 8y^3$$
$$27a^3 + 8b^3 = (3a + 2b)(9a^2 - 6ab + 4b^2)$$

5. Zbroji algebarske razlomke i dobiveni rezultat skрати:

A. $\left(a + b - \frac{2ab}{a + b}\right) : \left(\frac{a - b}{a + b} + \frac{b}{a}\right)$

B. $\left(\frac{a}{a + 1} - \frac{a^2}{a^2 - 1}\right) : \left(\frac{a^2}{a + 1} - \frac{a^3}{a^2 + 2a + 1}\right)$

Rješenje:

A.

$$\begin{aligned} \left(a + b - \frac{2ab}{a + b}\right) : \left(\frac{a - b}{a + b} + \frac{b}{a}\right) &= \frac{(a + b)^2 - 2ab}{a + b} : \frac{a(a - b) + b(a + b)}{a(a + b)} \\ &= \frac{a^2 + 2ab + b^2 - 2ab}{a + b} : \frac{a^2 - ab + ab + b^2}{a(a + b)} \\ &= \frac{a^2 + b^2}{a + b} \cdot \frac{a(a + b)}{a^2 + b^2} = a \end{aligned}$$

B.

$$\begin{aligned} \left(\frac{a}{a + 1} - \frac{a^2}{a^2 - 1}\right) : \left(\frac{a^2}{a + 1} - \frac{a^3}{a^2 + 2a + 1}\right) &= \frac{a(a - 1) - a^2}{(a - 1)(a + 1)} : \frac{a^2(a + 1) - a^3}{(a + 1)^2} \\ &= \frac{a^2 - a - a^2}{(a - 1)(a + 1)} : \frac{a^3 + a^2 - a^3}{(a + 1)^2} \\ &= \frac{-a}{(a - 1)(a + 1)} \cdot \frac{(a + 1)^2}{a^2} \\ &= -\frac{a + 1}{a(a - 1)} \end{aligned}$$