Prva školska zadaća - ISPRAVAK
(GT, HTT)

1. Za polinome $P(x) = 2x^2 + 3x - 2$ i $Q(x) = 2x^2 - 2x - 1$ odrediti:

$$P(x) + Q(x), P(x) - Q(x) P(x) \cdot Q(x)$$

2. Koristeći se formulama za kvadrat zbrojai razlike izračunaj:

   (a) $3(a^2 - 2) + (2a + 1)^2 - (3 - a)^2$
   (b) $(x + 2)^3 - (x - 2)^3 + 4(x^3 - 2)$

3. Zbroji algebarske razlomke i dobiveni rezultat skrati:

$$\frac{a - 6}{4 - a^2} + \frac{2}{2a - a^2} =$$

4. Izvrši naznačene operacije:

$$\left(\frac{a}{(a-2)^2} + \frac{8}{2a - a^2}\right) \cdot \left(1 + \frac{2}{a - 4}\right)^2 =$$

5. Zapiši u obliku $f(x) = g(x) \cdot q(x) + r(x)$ ako su zadani polinomi

$$f(x) = x^4 - 3x^3 - x^2 + x + 6; \quad g(x) = x^2 - 3x + 1$$

Rješenje:

1. Za polinome $P(x) = 2x^2 + 3x - 2$ i $Q(x) = 2x^2 - 2x - 1$ odrediti:

$$P(x) + Q(x), P(x) - Q(x) P(x) \cdot Q(x)$$

Rješenje:

$$\begin{aligned}
P(x) + Q(x) &= 2x^2 + 3x - 2 + 2x^2 - 2x - 1 = 4x^2 + x - 3 \\
P(x) - Q(x) &= 2x^2 + 3x - 2 - \left(2x^2 - 2x - 1\right) \\
&= 2x^2 + 3x - 2 - 2x^2 + 2x + 1 = 5x - 1 \\
P(x) \cdot Q(x) &= \left(2x^2 + 3x - 2\right) \cdot \left(2x^2 - 2x - 1\right) \\
&= 4x^4 - 4x^3 - 2x^2 + 6x^3 - 6x^2 - 3x - 4x^2 + 4x + 2 \\
&= 4x^4 + 2x^3 - 12x^2 + x + 2
\end{aligned}$$

2. Koristeći se formulama za kvadrat zbrojai razlike izračunaj:

   (a) $3\left(a^2-2\right)+(2a+1)^2-(3-a)^2$

   Rješenje:

$$\begin{aligned} 3\left(a^2-2\right)+(2a+1)^2-(3-a)^2 &= 3a^2-6+4a^2+4a+1-\left(9-6a+a^2\right) \\ &= 7a^2+4a-5-9+6a-a^2 \\ &= 6a^2+10a-14 \end{aligned}$$

   (b) $(x+2)^3-(x-2)^3+4\left(x^3-2\right)$

   Rješenje:

$$\begin{aligned} (x+2)^3-(x-2)^3+4\left(x^3-2\right) &= x^3+6x^2+12x+8-\left(x^3-6x^2+12x-8\right)+ \\ &= 5x^3+6x^2+12x-x^3+6x^2-12x+8 \\ &= 4x^3+12x^2+8 \end{aligned}$$

3. Zbroji algebarske razlomke i dobiveni rezultat skrati:

$$\frac{a-6}{4-a^2}+\frac{2}{2a-a^2}=$$

   Rješenje:

$$\begin{aligned} \frac{a-6}{4-a^2}+\frac{2}{2a-a^2} &= \frac{a-6}{(2-a)(2+a)}+\frac{2}{a(a-2)} \\ &= \frac{a(a-6)+2(2+a)}{a(2-a)(2+a)}=\frac{a^2-6a+4+2a}{a(2-a)(2+a)} \\ &= \frac{a^2-4a+4}{a(2-a)(2+a)}=\frac{(a-2)^2}{a(2-a)(2+a)} \\ &= \frac{(2-a)^2}{a(2-a)(2+a)}=\frac{2-a}{a(2+a)} \end{aligned}$$

4. Izvrši naznačene operacije:

$$\left(\frac{a}{(a-2)^2}+\frac{8}{2a-a^2}\right)\cdot\left(1+\frac{2}{a-4}\right)^2=$$

Rješenje:

$$\begin{aligned}
\left(\frac{a}{(a-2)^2}+\frac{8}{2a-a^2}\right)\cdot\left(1+\frac{2}{a-4}\right)^2 &= \left(\frac{a}{(a-2)^2}+\frac{8}{a(2-a)}\right)\cdot\left(\frac{a-4+2}{a-4}\right)^2 \\
&= \left(\frac{a}{(a-2)^2}-\frac{8}{a(a-2)}\right)\cdot\left(\frac{a-2}{a-4}\right)^2 \\
&= \frac{a^2-8(a-2)}{a(a-2)^2}\cdot\frac{(a-2)^2}{(a-4)^2} \\
&= \frac{a^2-8a+16}{a(a-2)^2}\cdot\frac{(a-2)^2}{(a-4)^2} \\
&= \frac{(a-4)^2}{a(a-2)^2}\cdot\frac{(a-2)^2}{(a-4)^2}=\frac{1}{a}
\end{aligned}$$

5. Zapiši u obliku $f(x)=g(x)\cdot q(x)+r(x)$ ako su zadani polinomi

$$f(x)=x^4-3x^3-x^2+x+6;\quad g(x)=x^2-3x+1$$

Rješenje: $q(x)=x^2-2,\quad r(x)=-5x+1$