

Rješenje: Promatrat ćemo omjer(količnik)

$$\begin{aligned}
 \frac{1 - x^{n+1}}{1 - x^n} &= \frac{1 - x^n + x^n - x^{n+1}}{1 - x^n} \\
 \frac{1 - x^n + x^n - x^{n+1}}{1 - x^n} &= \frac{1 - x^n}{1 - x^n} + \frac{x^n(1 - x)}{1 - x^n} = 1 + \frac{x^n(1 - x)}{1 - x^n} = \\
 &= 1 + \frac{x^n(1 - x)}{(1 - x)(1 + x + x^2 + \dots + x^{n-1})} = \\
 &= 1 + \frac{x^n}{(1 + x + x^2 + \dots + x^{n-1})} = \\
 &= 1 + \frac{1}{\frac{1}{x^n} + \frac{1}{x^{n-1}} + \frac{1}{x^{n-2}} + \dots + \frac{1}{x^2} + \frac{1}{x}} < 1 + \frac{1}{n}
 \end{aligned}$$

Ovo vrijedi jer je $0 < x < 1$, te je svaki od razlomaka $\frac{1}{x}, \frac{1}{x^2}, \dots, \frac{1}{x^n}$ veći od 1. Sada konačno imamo

$$\begin{aligned}
 \frac{1 - x^{n+1}}{1 - x^n} &< 1 + \frac{1}{n} = \frac{n+1}{n} \\
 \frac{1 - x^{n+1}}{1 - x^n} &< \frac{n+1}{n} \cdot \frac{1 - x^n}{n+1} \\
 \frac{1 - x^{n+1}}{n+1} &< \frac{1 - x^n}{n}
 \end{aligned}$$